Optimal Vibration Control and Innovization for Rectangular Plate

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***Abstract*—Vibration control of flexible structures has always been one of the most important issues and Among variant available control methods, active vibration control using piezoelectric sensors and actuators has become popular due to its high efficiency and flexibility for designing a control system. The main concern in designing a control system with piezoelectric patches is finding best position for patches. On the other hand, number of used sensors and actuators is another important issue which affects the costs of the project as well as the performance. The main goal of the present study is to control oscillation of a rectangular plate using minimum number of piezoelectric sensors and actuators (i.e., objective one) and finding their optimum placement to get the maximum possible performance (i.e., objective two); the mentioned two objectives are in conflict. The plate have been mathematically modeled using the Kirchhoff-Love theory. By considering the piezoelectric sensor-actuators effects, the control equation of the cantilever plate has been obtained. In order to find the optimum number and placement of the sensors and actuators, the multi-objective genetic algorithm (GA) has been used and the objective functions have been defined based on maximization of observability and countability indexes of the cantilever plate. After conducting the optimization process, a few thumb rules have been extracted using the innovization technique. The results have been verified by implementing the designed controller using the optimum solution found by optimization method. The importance of the rules found by innovization technique have been illustrated in the numerical discussion.**

***Index Terms*—Vibration Control, Kirchhoff-Love Plate, Fuzzy Logic Controller, Multi-objective Optimization, Genetic Algorithm, Evolutionary Computation, Innovization.**

I. Introduction

During the recent years, many researchers have focused on finding optimum solutions to control vibration of the flexible structures. Among various available vibration control methods, active vibration control using sensors and actuators has attracted attention due to its high efficiency for controlling vibration of flexible structures. Recent developments in piezoelectric materials and their applications such as distributed sensors and actuators in the field of control and vibration suppression [1] have been drawn high attention and resulted in a dense literature in the usage of piezoelectric patches in vibration control of the flexible structures. Since the number of sensors and actuators is often limited by physical or economical constraints, therefore their placement is fundamentally important to get a desirable performance for designed control system [2]. Optimal placement of piezoelectric sensors and actuators for vibration control of a composite cantilever plate has been studied by Qiu et al. [1]. They used the genetic algorithms to find efficient locations of piezoelectric sensors and actuators. They defined their objective function in a way that it maximized the observability and countability indexes. They also designed an efficient control method by combining Positive Position Feedback (PPF) and proportional-derivative control for vibration reduction. Their results showed that the presented control method was feasible and the optimal placement method was effective. In another the optimal placement of collocated angular rate sensors and Control Moment Gyroscope (CMG) actuators for a constrained gyroelastic body using genetic algorithms was investigated by Jia et al. [3]. They showed that the number of CMG.s embedded in the constrained flexible plate was not “the more, the better” for vibration suppression. Their results also showed that CMG.s were mainly placed at the corner and the two sides of the constrained plate. Chhabra et al. worked on optimal placement piezoelectric actuators on plate using Modified Control Matrix and Singular Value Decomposition (MCSVD) [4] . They considered the singular values of control matrix of ten actuators as fitness function and by maximizing it, they obtained the optimum placement of the actuators. They used GA and their results indicated that the position of the patches were symmetric to the center axis. The goal of the current study is to find the optimum number of piezoelectric sensors and actuators and their optimum position to design the control system with maximum controllability

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and observability. The genetic algorithm has been used by considering natural number coding for the numbers of the actuators and continues modeling for their positions on the plate. The plate is assumed to be rectangular and having boundary conditions of one fixed side and three free sides (i.e., cantilever plate). The H_2 norm [5] has been used to define controllability index as one of the objective functions. Constraints have been specified to prevent overlapping of the piezoelectric patches. Using the innovization concept, few design principles have been derived which can be used as thumb rules by practitioners. The obtained results have been verified by controller designed based on the optimum number and positions of the sensors and actuators. At the end, the results have been discussed in details.

II. Mathematical Modeling

1) Partial Differential Equation of Motion of The Plate: In this section, mathematical modeling of the system is defined. Fig. 1 shows the schematic model of the cantilever plate with general positions of the piezoelectric patches.

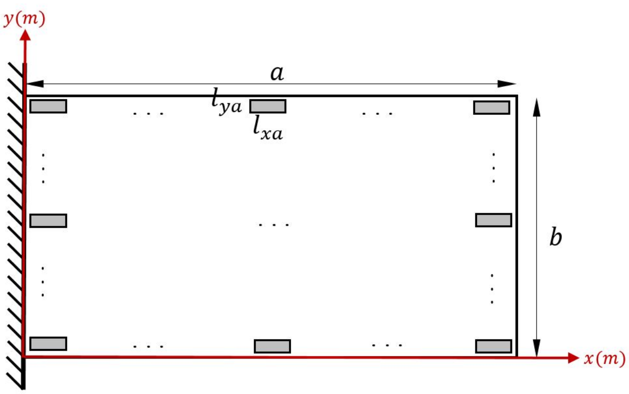

Fig. 1. Schematic model of the cantilever plate with general positions of piezoelectric patches

The partial differential equation of motion of a rectangular cantilever plate based on the Kirchhoff-Love plate theory can be written as follow:

$$D_p(\frac{\partial^4}{\partial x^4}+2\frac{\partial^4}{\partial x^2 y^2}+\frac{\partial^4}{\partial y^4})w(x,y,t)+\rho_p h_p\frac{\partial^2 w(x,y,t)}{\partial t^2}=0, \quad (1)$$

where $w(x, y, t)$ is the modal displacement of the rectangular plate, and D_p is the flexural rigidity which for an isotropic plate can be defined as:

$$D_p=\frac{E_p h^3}{12(1-\nu_p^2)} \quad (2)$$

In which E_p, ν_p, ρ_p, and h_p are Young's modulus, Poisson's ratio, mass density and thickness of the plate, respectively. x and y are the coordinate variables and t is the time parameter. The Galerkin's method has been utilized to find the approximate response of the Eq. (1). This method has been proved to predict the response of the system perfectly and has been widely used in the literature [6]. According to this method, the general form of the transverse deflection of the plate for the four first vibration modes can be defined as follow:

$$w(x,y,t)=\sum_{m=1}^{2}\sum_{n=1}^{2}W_{mn}(x,y)\phi_{mn}(t) \quad (3)$$

Where m and n denote the $(m, n)th$ vibration mode of the plate, $\phi_{mn}(t)$ represents the system time dependent modal coordinate, and $W_{mn}(x, y)$ is the corresponding modal displacement function in the x and y directions. By considering the cantilever (CFFF) boundary condition for the plate, $W_{mn}(x, y)$ can be defined as follow [7]:

$$W_{mn}(x,y)=X_m(x)Y_n(y), \quad (4)$$

where

$$X_m(x)=\cosh(\varepsilon_m x)-\cos(\varepsilon_m x)-\frac{\sinh(\varepsilon_m l)-\sin(\varepsilon_m l)}{\cosh(\varepsilon_m l)+\cos(\varepsilon_m l)} \times[\sinh(\varepsilon_m x)-\sin(\varepsilon_m x)] \quad (5)$$

$$Y_n(y)=\begin{cases}1 & n=1\\ \sqrt{3} & n=2\\ \sin(\varepsilon_n y)+\sin(\varepsilon_n y)+ \\ \frac{\cos(\varepsilon_n l)-\cosh(\varepsilon_n l)}{\sin(\varepsilon_n l)+\sinh(\varepsilon_n l)}[\cos(\varepsilon_n y)+ \\ \cosh(\varepsilon_n y)] & n\geq 3\end{cases} \quad (6)$$

2) Dynamic Analysis of Plate with Piezoelectric Patches: Attaching the piezoelectric patches will effect the dynamic behavior of the plate due to their physical characteristics. Such effects are too small, but not negligible, therefore it is necessary to consider these effects on the modal analysis of the system. As illustrated in Fig. 1, the rectangular piezoelectric sensors and actuators have been attached to the plate and will be used to measure the modal deflection and displacement velocity of the plate. Electrical circuit generated by sensors can be written as follow:

$$I_i(t)=-r_i\int_0^{l_{xai}}\int_0^{l_{yai}}(e_{31i}\frac{\partial^3 w}{\partial x^2\partial t}+e_{32i}\frac{\partial^3 w}{\partial y^2\partial t}+ 2e_{36i}\frac{\partial^3 w}{\partial x\partial y\partial t})\,dy\,dx, \quad (7)$$

where r_i shows the distance between middle plane of the ith sensor and the middle plane of the plate. e_{31i}, e_{32i}, and e_{36i} denote the piezoelectric stress constraints of each sensor. l_{xai} and l_{yai} are the length of each sensor patch in x and y directions, respectively. Also, the piezoelectric actuator coefficient could be obtained using the following relationship:

$$Piezo_{mn}^{i} = \frac{-1}{V_i} \int_0^a \int_0^b \Bigg(C_0^i \epsilon_{pe}^i \Big([\delta'(x - x_{1i}) - \delta'(x - x_{2i})] \times [H(y - y_{1i}) - H(y - y_{2i})] + [\delta'(y - y_{1i}) - \delta'(y - y_{2i})] \times [H(x - x_{1i}) - H(x - x_{2i})] \Big) + 2C_0^i \epsilon_{pe6}^i \times [\delta'(x - x_{1i}) - \delta'(x - x_{2i})] \times [\delta'(y - y_{1i}) - \delta'(y - y_{2i})] \; W_{mn}(x, y) \Bigg) \, dy \; dx \tag{8}$$

Considering the dynamic effect of the piezoelectric patches on the plate, one could derive the equation of motion of the plate with the attached piezoelectric patches as follow:

$$D_p \nabla^4 w + \mu \dot{w} + \rho_p h \ddot{w} + \sum_{i=1}^{N_a} \Big\{ C_0^i \epsilon_{pe}^i [\delta'(x - x_{1i}) - \delta'(x - x_{2i})] \times [H(y - y_{1i}) - H(y - y_{2i})] + C_0^i \epsilon_{pe}^i [\delta'(y - y_{1i}) - \delta'(y - y_{2i})] \times [H(x - x_{1i}) - H(x - x_{2i})] + 2C_0^i \epsilon_{pe6}^i [\delta'(x - x_{1i}) - \delta'(x - x_{2i})] \times [\delta'(y - y_{1i}) - \delta'(y - y_{2i})] \Big\} = 0 \tag{9}$$

3) Modal Analysis and State-Space Equations: By performing modal analysis on the partial differential equation of motion of the system, Eq. (10), and doing some mathematical simplification, the ordinary differential equations (ODE) for the ith mode can be derived as:

$$\ddot{\phi}_i + \omega_i^2 \phi_i + \omega_i \zeta_i \dot{\phi}_i + \frac{\sum_{j=1}^{N_a} Piezo_i^j}{M_i} = 0, \tag{10}$$

where ω_i is the natural frequency of the $i\ th$ mode, ζ_i is the damping coefficient of the $i\ th$ mode of the system. Having ODE of each mode, one can rewrite the ODEs in standard state-space form as:

$$\dot{z} = Az + B_p u_p \tag{11}$$

$$Y = C_p z \tag{12}$$

III. Problem Definition

In this part, the optimizing problem is defined based on the controllability and observability of the control equations defined in the previous section. Finding the optimum number of piezoelectric and their optimum position is the goal of the optimization problem. The objective functions will be defined and some constraints will be considered which will be explained in details in the following subsections.

1) Controllability Index: In order to find the optimal placement of sensors and actuators, the H_2 norm of the control transfer function is used. Considering (A, B, C) as the system state-space representation, the transfer function would be defined as:

$$G(\omega) = C(j\omega I - A)^{-1} B \tag{13}$$

Therefore the norm of the transfer function can be found using the following relationship [5]:

$$||G||_2 = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{\infty} tr(G^*(\omega) G(\omega)) \; d\omega} \tag{14}$$

By substituting Eq. (13) into Eq. (14) and doing some mathematical simplifications, the H_2 norm function can be defined as function of state-space matrices as follow:

$$||G||_2 = \frac{|B_i| \times |C_i|}{2\sqrt{\zeta_i \omega_i}} \tag{15}$$

The placement index of each jth actuator for the ith mode of the system is defined as [1]:

$$\delta_{2ij} = w_i ||G||_{2ij} \quad ,i = 1, 2, ..., (m \times n), \quad j = 1, 2, ..., N_a. \tag{16}$$

Where w_i represents the weight of each mode which reflects the importance of that mode for controlling system. Therefore, the H_2 norm optimal placement index for the system based on the controllability can be defined as the first objective function of the optimization problem:

$$Max: \; \mu_j = \sqrt{\sum_{i=1}^{m \times n} \delta_{2ij}} \tag{17}$$

The other objective function of the optimization problem is the number of the actuators (or the total cost) which needs to be minimized:

$$Min: \; N_{pe} \equiv Cost(\$) = 200 \times N_{pe}, \quad 1 \le N_{pe} \le 20 \tag{18}$$

where N_{pe} is the number of piezoelectric patches.

According to the available literature, there are many different methods such as Goal Programming, NSGA-II [8], ϵ-constraints [9], etc. to find the Pareto front set of the multi-objective problem.

The problem can be solved as a bi-objective or bi-level optimization problem, but the challenging part is its variable dimension size, due to the change in number of piezoelectric patches. In order to tackle this problem, inspired from the ϵ-constraint method [9], for each number of piezoelectric patches, the genetic algorithm (GA) has been run for the single objective (i.e., performance) and the best solution has been recorded. By repeating this approach for all possible number of piezoelectric patches and combining all the solutions obtained from the GA, the Pareto-front solutions have been achieved, By this way, the problem is treated as N single-objective problems, where N is the number of cases considered for variant number of piezoelectric patches. The GA function of MATLAB® software is used to find the optimal solutions in this study which detailes of the algorithm can be found in the references [10]–[12]. The control parameters set for GA are the population size of 100 (N_p), mutation rate (C_m) of 0.01, crossover rate (C_r) of 0.8, and the termination condition is based on allowed maximum generation numbers which is mentioned in the Eq. (18). For each single-objective problem, the dimension of the problem is equal to the all x and y positions of the piezoelectric patches. For instance, if the number of the piezoelectric patches are 13, so that the dimension of the GA would be $D = 26$. Also, for each problem the maximum number of generations was defined based on its dimension as the termination condition $Max_{gen} = 500\ D$. It is noteworthy to mention that using optimization method to find the best solution reduces the computational time efficiently in comparison with the brute force search which is computationally expensive. This is while number of available positions for each piezoelectric patch is 768, obtained from $N_x \times N_y$ with $N_x = 24$ and $N_y = 32$. Therefore the total number of different candidate combinations can be calculated using the following formula:

$$C_{768,N_{pe}} = \binom{768}{N_{pe}} = \frac{768!}{N_{pe}!(768 - N_{pe})!} \tag{19}$$

For example, as the number of piezoelectric patches increases from 1 to 5, 10, 15 and 20, the number of possible combinations which results in expensive computational time for performing exhaustive search increase rapidly as follow:

$$\begin{array}{ll}
N_{pe} = 1 & C_{768,1} = 768 \\
N_{pe} = 5 & C_{768,5} = 2.1977 \times 10^{12} \\
N_{pe} = 10 & C_{768,10} = 1.8548 \times 10^{22} \\
N_{pe} = 15 & C_{768,15} = 1.2711 \times 10^{31} \\
N_{pe} = 20 & C_{768,20} = 1.6321 \times 10^{39}
\end{array}$$

Considering the time needed for calling the controllability index in each run of the program, it is obvious that performing the exhaustive search is almost impossible due to the high computation time. As an example, for $N_{pe} = 10$, the function call takes about 0.04 seconds and by considering number of possible combinations for 10 number of piezoelectric patches, approximately 1.2684×10^{13} years is needed to finish the search.

IV. Numerical Results and discussions

In this section, the numerical results have been discussed. The plate is considered with the length of $a = 1.5\ m$ and width of $b = 0.5\ m$. The physical characteristics of the plate are $E_p = 70\ GPa$, $\nu_p = 0.33$, $\rho_p = 2700\ kg/m^3$, and $h_p = 0.015\ m$. The Piezoelectric patches have the physical characteristics as $l_{xa} = 0.06\ m$, $l_{ya} = 1.5\ m$, $E_{pe} = 63\ GPa$, $\nu_{pe} = 0.3$, $\rho_{pe} = 7650\ kg/m^3$, and $h_{pe} = 0.001\ m$; which are length of the patches in x and y directions, Young's modulus, Poisson's ratio, mass density and the thickness of the patches, respectively. It is assumed that all piezoelectric patches are the same and attached to the plate horizontally.
The GA has been run for 20 times and in each time the termination criteria has been defined to reach the maximum number of generation defined for the problem in Eq. (18):

The solution found by GA in each step is the optimal positions for piezoelectric patches on the plate. As three case studies, the best found positions and GA performance plot have been shown in Fig. 2 to Fig. 8. It should be mentioned that each sell shows a candidate position and filled cells represent the positions found by GA as solution:

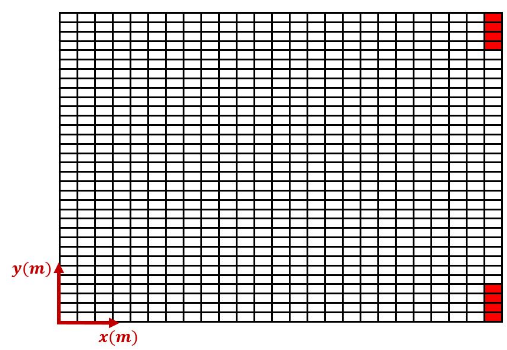

Fig. 2. Schematic model of the cantilever plate with optimum positions of piezoelectric patches for $N_{pe} = 8$.

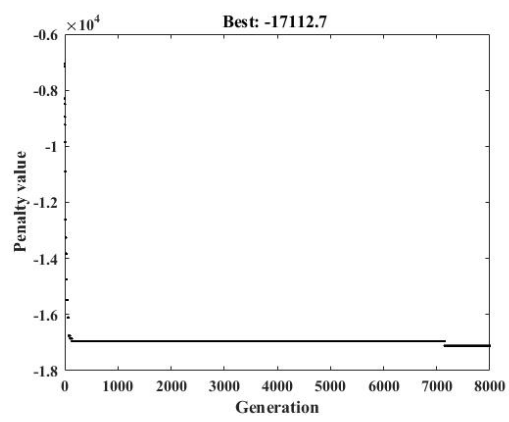

Fig. 3. GA performance plot for $N_{pe} = 8$.

1) Results Analysis: It can be seen that in the all shown case studies, the optimum positions found by the GA are mainly at the corners of the free end of the plate. Few number of the piezoelectric patches have been placed at the sides centers and almost no piezoelectric patches has been placed at the fixed end. Considering the vibration behavior of a cantilever plate [13], to be able to control the vibration, since the highest

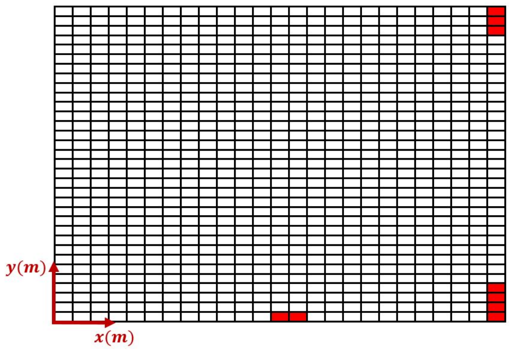

Fig. 4. Schematic model of the cantilever plate with optimum positions of piezoelectric patches for $N_{pe} = 9$.

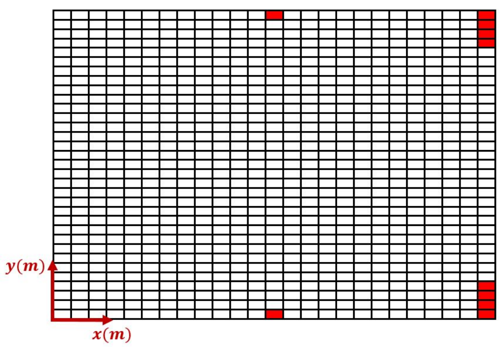

Fig. 5. Schematic model of the cantilever plate with optimum positions of piezoelectric patches for $N_{pe} = 10$.

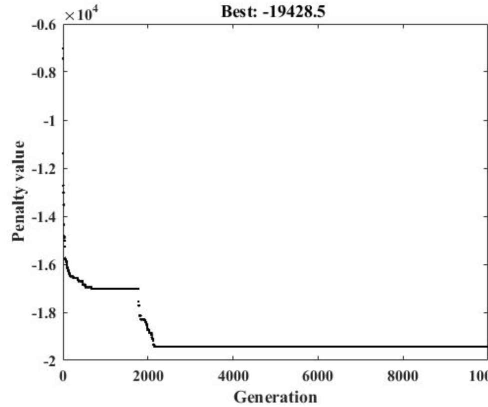

Fig. 6. GA performance plot for $N_{pe} = 10$.

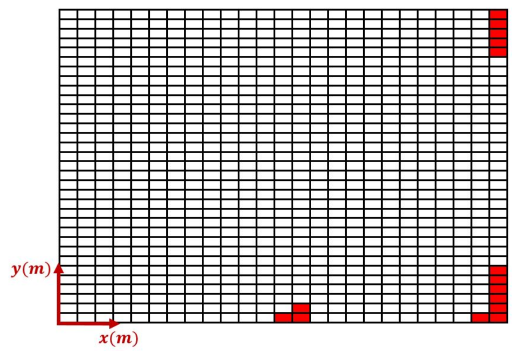

Fig. 7. Schematic model of the cantilever plate with optimum positions of piezoelectric patches for $N_{pe} = 15$.

displacement occurs at the free ends, it is logical to set majority of the piezoelectric patches at the free end to be able to control the vibration faster.

In each run, the average of the best solutions so far and also the best solution have been collected and the Pareto-front (PF) has been created as shown in Fig. 10. This PF illustrates the best solutions found by the optimization for the objective functions contrallability index and number of piezoelectric patches (cost). This should be mentioned that the cost has been calculated based on the number of piezoelectric patches used in each run (200 $ per piezoelectric patch).

As shown in Fig. 10, there are two important bend (i.e., knee points) happening at $N_{pe} = 9$ and $N_{pe} = 14$. The controllability index increases by 1.89% when the number of actuators increases from 8 to 9, while by adding only one more actuator, the controllability increases by 11.94%. The same is happening for point $N_{pe} = 14$, when the designer increases the number of actuators from 14 to 15, adding only one more actuator will lead to 14.76% increase in controllability index. In order to make it easier for a decision maker to select one of the solutions, Table I is presented for each optimal solution of the PF shown in Fig. 10.

In order to find the closest possible PF to the optimal PF, a warm initialization has been performed with 500 and 3000 generations in each run. The warm initialization has been applied by feeding the GA obtained PF solutions shown in Fig. 10 as an initial population. The results were the same except for $N_{pe} = 18$ and $N_{pe} = 19$ in which 0.84% and 0.2% improvements have been obtained, respectively. The obtained PFs as results of the warm initialization have been shown in Fig. 10.

V. Extracting Design Principles After Optimization Using Innovization

Innovization is a recently introduced methodology in which the solutions found by optimization are analyzed to extract useful relationships and design principles. These design principles can help designers and practitioners to obtain deeper understanding of the problem and also motivate them for working on further applications and solving more complex problem [14]–[17]. Inspired by innovization technique and by analysing found optimal positions of sensor-actuator, the following thumb rules can be derived which are usable by designers:

- Rule 1: Positions should be chosen in a symmetrical style.
- Rule 2: The corners of the free end of the plate should be the very first positions to be chosen.
- Rule 3: After free corners (for $N_{pe} > 8$ in this study), the middle positions of the side edges should be filled with the piezoelectric patches.

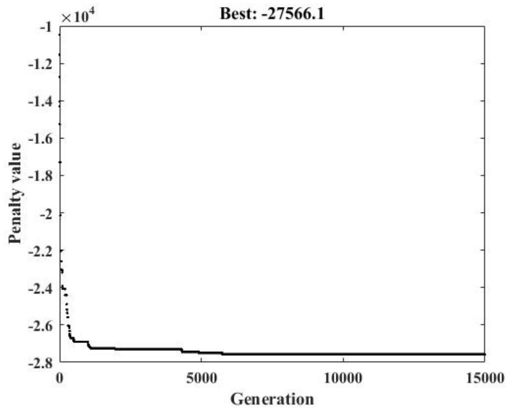

Fig. 8. GA performance plot for $N_{pe} = 15$.

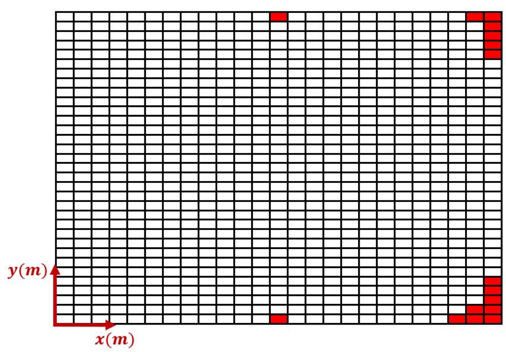

Fig. 9. Schematic model of the cantilever plate with optimum positions of piezoelectric patches for $N_{pe} = 16$.

Rule 4: Positions at the fixed ends should not be chosen unless no other position is available.

Rule 5: It is better not to choose positions in the middle of the plate.

Considering the first mentioned rule, the effect of the symmetric positions has been studied by changing the obtained positions by the optimization in a symmetric style. Therefore, by combining the artificial and human intelligence, symmetric positions have been chosen for the piezoelectric patches and some of them have been shown in Fig. 12 and Fig. 13.

By taking into account the modified symmetric positions, the new PF has been compared with the PF found by the optimization in the previous section. It can be seen that in the all cases, except one ($N_{pe} = 15$), the controllability index has increased and even some cases which had been dominated in the previous PF, $N_{pe} = 12$ and $N_{pe} = 16$, now can be seen in the PF, Fig. 14. The reason behind the improvements in PF after applying the innovization based rules is that the evolutionary algorithms performance in local search is week and innovization acts like a local search for the solution found by the evolutionary algorithm.

VI. Verification of Vibration Control

In order to illustrate how significant is to find the optimum positions for the piezoelectric patches, solution of the optimization for ten number of patches has been fed into a Fuzzy Logic Controller designed and simulated by SIMULINK® software. In the fuzzy controller, the goal is to quickly damp vibration of the cantilever plate. In order to aim this goal, the sensors have been assigned to detect the plate vibration velocity and the actuators are responsible for applying external forces to cancel out the vibration of the plate. Membership functions for input and output of the fuzzy controller have been considered in five levels as high negative (HN), negative (N), zero, positive (P), and high positive (HP) for the velocity and the actuation force. Five fuzzy rules have also been defined to apply high positive force while the velocity is high negative or apply high negative force when the velocity is high positive. As numerical investigation, two cases have been studied: one using the optimum positions found by the optimization algorithm, and the other case utilizing randomly placed piezoelectric patches on the plate. As it was mentioned in the previous section, the symmetry of the patches position is one of the most important rules obtained based on the innovization

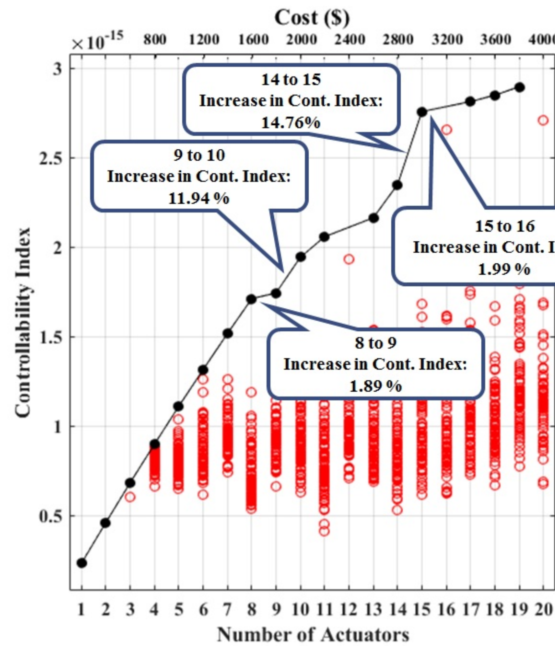

Fig. 10. Pareto front, Controllabilty index, Number of Actuators, and Cost (200$ per patch).

TABLE I
Table of Pareto-front

Num. Piezo	Contr. index ($\times 10^{-16}$)	cost($)
1	2.344	200
3	6.836	600
5	11.1	1000
7	15.16	1400
9	17.44	1800
11	20.58	2200
13	21.63	2600
15	27.57	3000
17	28.13	3400
19	28.96	3800

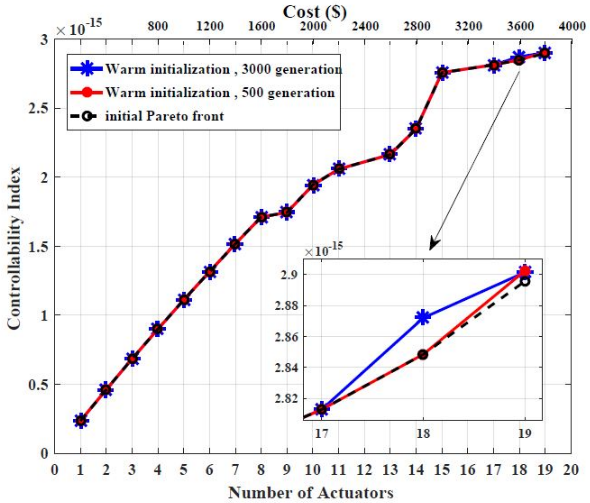

Fig. 11. Warm initialization considering different number of generations, 500 and 3000

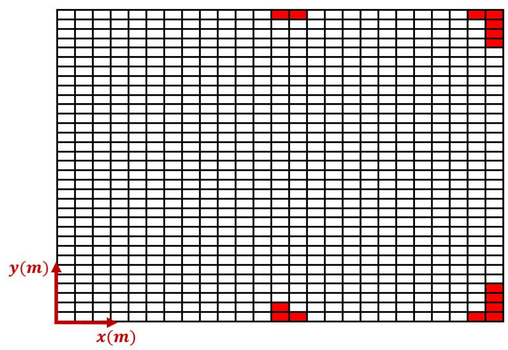

Fig. 12. Modified positions based on symmetry ($N_{pe} = 15$)

and one may predict that by changing the positions from the optimum symmetric ones into the randomly distributed ones, the controller performance will extremely decrease.
As shown in Fig. 15 and Fig. 16, when the piezoelectric patches have been placed in the optimum positions, the vibration of the plate is controlled within shorter time in all four first vibration modes of the system. Such numerical results are strong proofs that show the efficiency of the design rules found by innovization.

VII. Conclusion Remarks

Active vibration control of the rectangular isotropic plate using piezoelectric sensors and actuators with optimum number and positions have been performed in this study. Multiobjective optimization problem was defined to get the best possible solutions and to find the PF. A genetic algorithm inspired by ϵ-constraint method have been used to find the PF. In order to get the optimal positions for sensors and actuators, genetic algorithm was used and the objective function was defined as the H_2 norm of the controllability of the plate with piezoelectric patches. A result analysis was performed on the PF and was

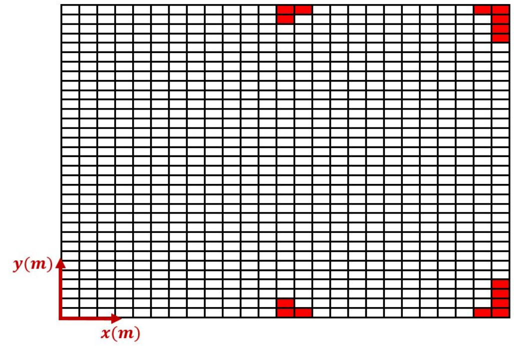

Fig. 13. Modified positions based on symmetry ($N_{pe} = 16$)

shown that sometimes adding one more piezoelectric patch increases the controllabilty with high percentage (it happens in knee points of PF). The optimal positions found by GA were illustrated for different case studies and the GA performance plots were shown in the result part. Based on the innovization technique, few useful design principles were learned. One of the most important of these rules says that to get the highest controllability, the position of piezoelectric patches should be chosen symmetrically. Also, the free end corners should be the very first positions to be chosen by the designer. Choosing symmetric positions for piezoelectric patches (the first rule learned by innovization) changed the PF in a way that some of the dominated points in the initial PF become evident after applying the rule. The fuzzy logic controller was utilized to investigate the vibration control of the first forth mode of the plate. The results showed that using the optimal positions for the placement of the sensors and actuators highly decreases the vibration control time efficiently.

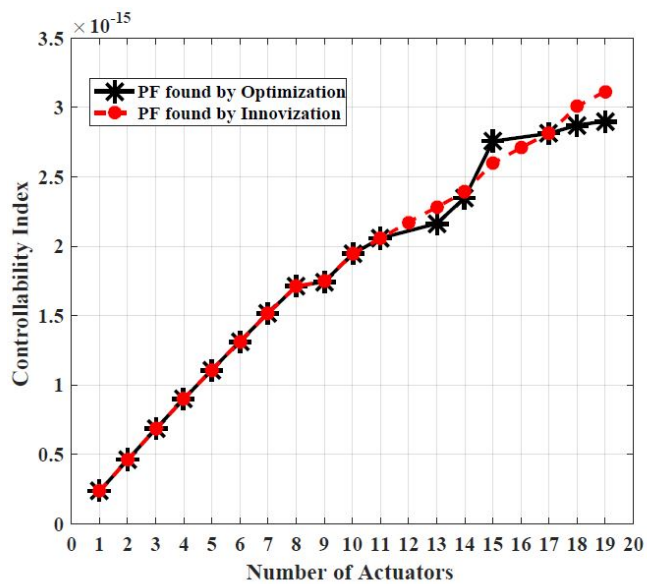

Fig. 14. The PF found by optimization vs. PF found by Innovization.

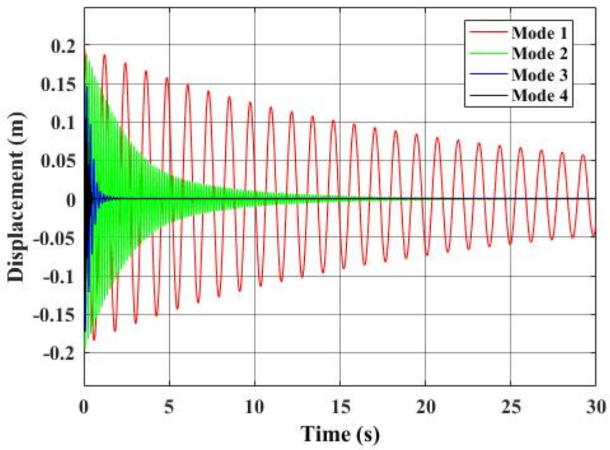

Fig. 15. Vibration control with optimum solution

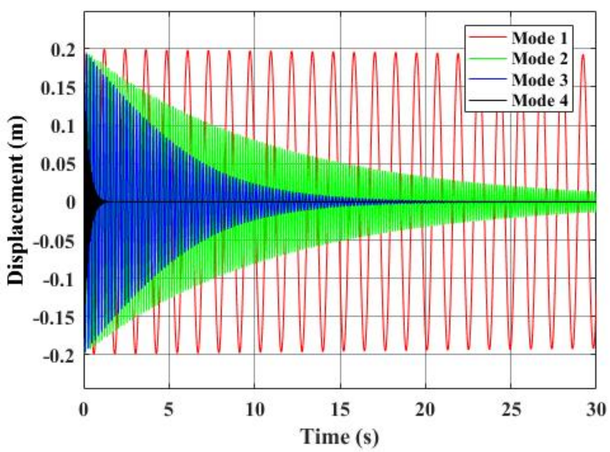

Fig. 16. Vibration control without optimum solution

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